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Super Elevation and Radius of Curvature of Bent Channels

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ABSTRACT
Increasing erosion and altering the morphology of the channel. If meanders are not properly stabilized, they may continue to develop and shift. This paper explains the physics of flows around bends. Scouring on the concave banks will result from the secondary flood pattern created by the ensuing curve flow. This movement will contribute to the meanders' increased curvature when combined with silt depositions on the convex banks. Centrifugal forces, secondary currents, and the physical properties of the channel interact intricately to affect the flow of water around bends in open channels. These elements are crucial to take into account in both natural and artificial streams since they affect the flow velocity, sediment transport, erosion, and deposition patterns leading to the accretion of sediments called point bars. Channels designs are usually advised to be subcritical and uniform in flow with Froude number less than 0.6 to avoid standing waves and are guided against critical depth. The freeboard or height of the concave edge or outer bank should be raised by 70-percent of the super elevation. The channel should be designed for average flow velocity of 0.7m/s suppose the discharge carries sediments to circumvent sedimentation in the system. Riprap and vegetation are examples of channel engineering projects that can be utilized to stabilize the banks and stop excessive erosion.

Keywords: *Morphology, Point bars, Centrifugal forces, Super elevation, Concave bank, Convex bank, Scouring*

INTRODUCTION

Background of the Study

Channels are usually designed for subcritical, uniform flow and are guided against critical depth. Freeboard is the clearance vertical gap between the top level of the channel bank and the designed water surface. Meandering river are formed by series of bends with their outer (concave) banks scoured as a result of higher velocities and deposits bed loads towards the inner (convex) banks. Several important hydrodynamic concepts are involved in the complicated water flow around a bend in an open channel, such as a river, stream, or man-made channel. The curvature of the channel creates a centrifugal force when water flows around a bend. This results in a difference in velocity between the bend's inner and outer banks as the water moves outward from the bend's center. Generally speaking, the inner bank has a lower velocity and the outer bank a higher one. Centrifugal forces, hydrostatic pressure force differences and secondary currents will continue the bend-forming meander process that occurs when a river reaches barrier.

Scouring on the concave banks will result from the secondary flood pattern created by

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the ensuing curve flow. This movement will contribute to the meanders' increased curvature when combined with silt depositions on the convex banks. The water surface will have a variable elevation (super elevation) at the bend due to the concomitant centrifugal force, with higher levels at the concave banks than at the convex banks. The super-elevation between the inner and outer banks, which results in a difference in the hydrostatic pressures between the opposing sides of the control volume, is responsible for the net pressure force in the radial direction. The balance of radial forces acting on the fluid column can be used to determine the super elevation in the curve channel. The pressure forces counteract the centrifugal forces brought on by the transverse water surface inclination [1-6].

Secondary currents are produced as the water passes through the bend. These are other spiral flow patterns that travel from the outer bank to the inner bank and then to the water's surface. Often referred to as "helical flow," this spiraling flow produces turbulence, which causes sediment to be eroded on the outer bend and deposited on the inner bend. Erosion of the bank may result from increased shear stress caused by the faster-moving water on the bend's outer edge. Features like cut banks may result from this approach. **Point bars** which are collections of bed loads, sands and/or gravels, are created as silt settles out due to the slower-moving water on the inner side. This causes the channel to gradually move outward from the inner bank [2].

Figure 1 shows a meandering river with sediment accretion at the inner bank.

Fig. 1: *Meandering River with Point Bar*
(Source: [2])

Compared to straight reaches, bends in an open channel increase flow resistance, which results in energy losses. Usually, these losses are greater in the vicinity of the

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bend's sharpest point. To overcome the frictional forces from the banks and channel bed, the flow could need more energy. Over time, bends in natural streams can result in meandering flow patterns. If meanders are not properly stabilized, they may continue to develop and shift, increasing erosion and altering the morphology of the channel. Riprap and vegetation are examples of channel engineering projects that can be utilized to stabilize the banks and stop excessive erosion.

METHODOLOGY

Radius of Curvature of Motion of a Particle in a Curved Path

Figure 2 shows the trajectory of a particle in motion through a curved path.

Fig. 2: *Particle in Motion through a Curved Trajectory*

$$ds = \rho d\theta \quad (1)$$

$$ds = \sqrt{dx^2 + dy^2} \quad (2)$$

$$ds = dx\sqrt{1 + \left(\frac{dy}{dx}\right)^2} \quad (3)$$

Curvature, $\mathcal{K}$:

$$\mathcal{K} = \frac{1}{\rho}$$

But $\frac{dy}{dx} = tan\theta$

Differentiating $\frac{dy}{dx} = tan\theta$ w.r.t. "x", we have:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(tan\theta)$$

$$\frac{d^2y}{dx^2} = sec^2\theta\frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = (1 + tan^2\theta)\frac{d\theta}{dx}$$

$$\frac{d\theta}{dx} = \frac{1}{1 + tan^2\theta}\frac{d^2y}{dx^2} \quad (4)$$

Now, dividing eqn. (1) by (3), we have:

$$1 = \frac{\rho}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}\frac{d\theta}{dx}$$

$$\frac{d\theta}{dx} = \frac{1}{\rho}\sqrt{1 + \left(\frac{dy}{dx}\right)^2} \quad (5)$$

Equating eqn. (4) and (5), we have:

$$\frac{1}{\rho}\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \frac{1}{1 + tan^2\theta}\frac{d^2y}{dx^2}$$

$$\rho = \frac{(1 + tan^2\theta)\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}{\frac{d^2y}{dx^2}}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}{\frac{d^2y}{dx^2}}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Hence,

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$$\boldsymbol{\rho} = \frac{\left[\mathbf{1} + \left(\frac{\boldsymbol{dy}}{\boldsymbol{dx}}\right)^{\mathbf{2}}\right]^{\mathbf{3/2}}}{\left|\frac{\boldsymbol{d^2y}}{\boldsymbol{dx^2}}\right|} \qquad (\mathbf{6})$$

Centripetal Acceleration in a Curved Path

Recall,

$$v = \rho\dot{\theta} \qquad (7)$$

We know, in vector notations:

$$\bar{v} = v\hat{u}_T \qquad (8)$$

$$\bar{a} = \frac{d}{dt}(v\hat{u}_T)$$

$$\bar{a} = \dot{v}\hat{u}_T + v\dot{\hat{u}}_T \qquad (9)$$

But

$$\Delta\hat{u}_T = 1 \cdot \dot{\theta} \cdot \Delta t \cdot \hat{u}_n$$

$$\frac{\Delta\hat{u}_T}{\Delta t} = \dot{\theta} \cdot \hat{u}_n$$

$$\therefore \dot{\hat{\boldsymbol{u}}}_{\boldsymbol{T}} = \dot{\boldsymbol{\theta}}\hat{\boldsymbol{u}}_{\boldsymbol{n}}$$

Plug in $\dot{\hat{u}}_T = \dot{\theta}\hat{u}_n$ into eqn. (9), we have:

$$\bar{a} = \dot{v}\hat{u}_T + v\dot{\theta}\hat{u}_n \qquad (10)$$

But $\dot{\theta} = \frac{v}{\rho}$

Plug in $\dot{\theta} = \frac{v}{\rho}$ into eqn. (10), we have:

$$\bar{a} = \dot{v}\hat{u}_T + v\left(\frac{v}{\rho}\right)\hat{u}_n$$

$$\bar{\boldsymbol{a}} = \dot{\boldsymbol{v}}\hat{\boldsymbol{u}}_{\boldsymbol{T}} + \frac{\boldsymbol{v^2}}{\boldsymbol{\rho}}\hat{\boldsymbol{u}}_{\boldsymbol{n}} \qquad (\mathbf{11})$$

Assuming a constant velocity through the bend,

$$\rightarrow \dot{\boldsymbol{v}} = \mathbf{0}$$

$$\therefore \bar{\boldsymbol{a}} = \frac{\boldsymbol{v^2}}{\boldsymbol{\rho}}\hat{\boldsymbol{u}}_{\boldsymbol{n}} \qquad (\mathbf{12})$$

Centrifugal Acceleration of Flow in Curved Channels

Figure 3 shows the side view of an open curved channel of fluid flowing around the bend.

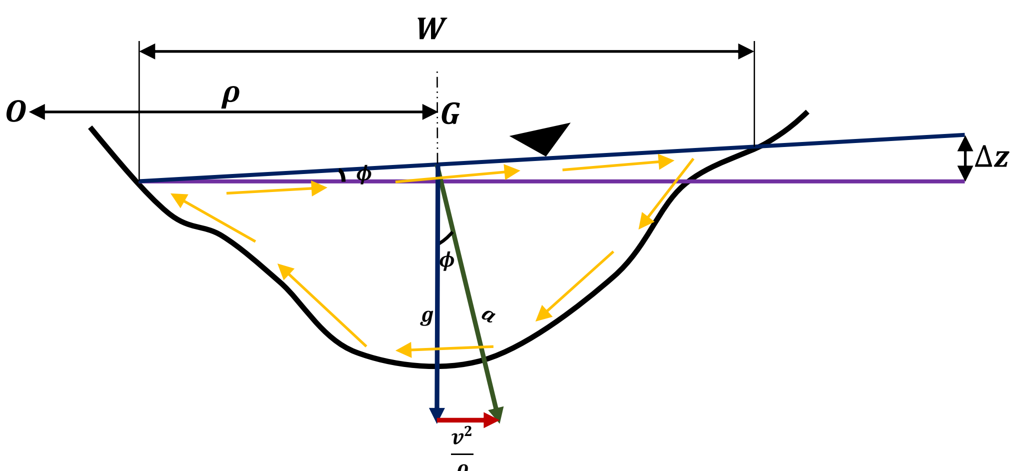

Fig. 3: *Open Channel Fluid Flow around a Concave-Convex Bends*

$$tan\phi = \frac{\left(\frac{v^2}{\rho}\right)}{g} = \frac{1}{g}\frac{v^2}{\rho}$$

$$tan\phi = \frac{\Delta z}{W}$$

$$\frac{\Delta z}{W} = \frac{1}{g}\frac{v^2}{\rho}$$

$$\therefore \Delta\boldsymbol{z} = \frac{\boldsymbol{W}}{\boldsymbol{g}}\frac{\boldsymbol{v^2}}{\boldsymbol{\rho}} \qquad (\mathbf{13})$$

Where, $W =$ Top width of the channel, $\Delta z =$ Super elevation around the concave edge, $\rho =$ Radius of curvature at any instant in time around the bend, $v =$ Average velocity of flow through the open channel.

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The height of the concave edge or outer bank should be raised by 70-percent of the super elevation (Δz). The channel should be designed for average flow velocity of 0.7m/s suppose the discharge carries sediments to circumvent sedimentation in the system [1].

CONCLUSION

Centrifugal forces, secondary currents, and the physical properties of the channel interact intricately to affect the flow of water around bends in open channels. These elements are crucial to take into account in both natural and artificial streams since they affect the flow velocity, sediment transport, erosion, and deposition patterns. Channels designs are usually advised to be subcritical and uniform in flow with Froude number less than 0.6 to avoid standing waves and are guided against critical depth. The height or freeboard of the concave edge or outer bank should be raised by 70-percent of the super elevation. The channel should be designed for average flow velocity of 0.7m/s suppose the discharge carries sediments to circumvent sedimentation in the system. Riprap and vegetation are examples of channel engineering projects that can be utilized to stabilize the banks and stop excessive erosion.

Nomenclature

W-Top width of the channel
Δz-Super elevation around the concave edge
ρ-Radius of curvature at any instant in time around the bend
$\mathcal{K}$-Curvature around the bend at time, t
v- Average velocity of flow through the open channel
$\hat{u}_T$-Tangential directional unit vector
$\hat{u}_n$-Normal directional unit vector

REFERENCES

1. Wurbs, R.A. & James, W.P. (2007). Water Resources Engineering, Prentice Hall of India, New Delhi-110001
2. Google (2025). Retrieved from: https://geo.libretexts.org/Courses/University_of_California_Davis/GEL_109%3A_Sediments_and_Strata_%28Sumner%29/Textbook_Construction/Meandering_Rivers
3. Abam, T. K. S. & Okagbue, C. O. (2000). Construction and performance of river bank erosion protection structure in the Niger Delta. *Bulletin of the Association of Engineering Geologists*, 23(4), 499-506.
4. Agabriel, O. & Bodensteiner, L.R. (2012). Impacts of riprap on wetland shorelines, upper Winnebago pool lakes,Wisconsin. *Wetlands*, 32(1), 105–117.
5. Leon, A. S. & D.Wre, P. E. (2023). Introduction to Sediment Transport in Open Channel Flows. Available at: https://web.eng.fiu.edu/arleon/courses/Open_Channel/Lectures/Sediment_Transport.pdf
6. Leo, F. & Rijn, V. H. (2016). Stability design of coastal structures 1 (seadikes, revetments, breakwaters and groins). *Coastal Engineering*, 11(1), 1-9.

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